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Intuitionistic Fuzzy a-Ideals of BCI-Algebras with Interval Valued Membership& Non Membership Functions

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Abstract: The purpose of this paper is to define the notion of an interval valued Intuitionistic Fuzzy a-ideal (briefly, an i-v IF a-ideal) of a BCI – algebra. Necessary and sufficient conditions for an i-v Intuitionistic Fuzzy a-ideal are stated. Cartesian product of i-v Fuzzy ideals are discussed.

I. INTRODUCTION

The notion of BCK-algebras was proposed by Imai and Iseki in 1996. In the same year, Iseki [6] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. Since then numerous mathematical papers have been written investigating the algebraic properties of the BCK/BCI-algebras and their relationship with other universal structures including lattices and Boolean algebras. Fuzzy sets were initiated by Zadeh[10]. In [9],Zadeh made an extension of the concept of a Fuzzy set by an interval-valued fuzzy set. This interval-valued fuzzy set is referred to as an i-v fuzzy set. In Zadeh also constructed a method of approximate inference using his i-v fuzzy sets. In Birwa's defined interval valued fuzzy subgroups of Rosenfeld`s nature , and investigated some elementary properties. The idea of "intuitionistic fuzzy set" was first published by Atanassov as a generalization of notion of fuzzy sets. After that many researchers considers the Fuzzifications of ideal and sub algebras in BCK/BCI-algebras. In this paper, using the notion of interval valued fuzzy set, we introduce the concept of an interval-valued intuitionistic fuzzy BCI-algebra of a BCI-algebra, and study some of their properties. Using an i-v level set of i-v intuitionistic fuzzy set, we state a characterization of an intuitionistic fuzzy a-ideal of BCI-algebra. We prove that every intuitionistic fuzzy a-ideal of a BCI-algebra X can be realized as an i-v level a-ideal of an i-v intuitionistic fuzzy a-ideal of X. in connection with the notion of homomorphism, we study how the images and inverse images of i-v intuitionistic fuzzy a-ideal become i-v intuitionistic fussy a-ideal.

II. PRELIMINARIES

Let us recall that an algebra (X,*,0) of type (2,0) is called a BCI-algebra if it satisfies the following conditions:1.((x*y)*(x*z))*(z*y)=0,2.(x*(x*y))*y=0,3.x*x=0,4.x*y=0 and y*x=0 imply x=y,for all x,y,z X. In a BCI-algebra, we can define a partial ordering"≤" by x≤y if and only if x*y=0.in a BCI-algebra X, the set M={x X/0*x=0} is a sub algebra and is called the BCK-part of X. A BCI-algebra X is called proper if X-M≠ϕ. otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:

1. (x*y)*z=(x*z)*y, 2.x*0=0, 3. x ≤y imply x*z ≤y*z and z*y ≤z*x, 4. 0*(x*y) = (0*x)*(0*y),
5. 0*(x*y) = (0*x)*(0*y), 6. 0*(0*(x*y)) =0*(y*x), 7. (x *z)*(y*z) ≤x*y

An intuitionistic fuzzy set A in a non-empty set X is an object having the form A= $\{<x,\mu_A(x),\upsilon_A(x)>/x \; X\}$,Where the functions $\mu_A : X\to[0,1]$ and $\upsilon_A: X\to[0,1]$ denote the degree of the membership and the degree of non membership of each element x X to the set A respectively, and $0\le \mu_A(x) + \upsilon_A(x) \le 1$ for all x X.Such defined objects are studied by many authors and have many interesting applications not only in the mathematics. For the sake of simplicity, we shall use the symbol $A= [\mu_A, \upsilon_A]$ for the intuitionistic fuzzy set $A= \{[\mu_A(x),\upsilon_A(x)]/ x \; X\}$.

Definition 2.1:A non empty subset I of X is called an ideal of X if it satisfies:1. 0 I,2.x*y I and y I $\Rightarrow$ x I.

Definition 2.2: A fuzzy subset μ of a BCI-algebra X is called afuzzy ideal of X if it satisfies:1.μ (0) ≥μ(x), 2. μ(x) ≥min {μ(x*y), μ(y)}, for all x,y X.

Definition 2.3: A non empty subset I of X is called a- ideal of X if it satisfies:1. 0 I. 2. (x*z)*(0*y) I and y I imply x*z I.Putting z=0 in(2) then we see that every a- ideal is an ideal.

Definition 2.4: A fuzzy set μ in a BCI-algebra X is called an fuzzy a- ideal of X if1.μ (0) ≥μ(x), 2. μ(y*x) ≥min {μ ((x*z)*(0*y)), μ (z)}.

Definition 2.5: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cap B$ membership function $\mu_{A \cap B}$ is defined by $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, x \in X$.

Definition 2.6: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cup B$ with membership function $\mu_{A \cup B}$ is defined by $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$.

Definition 2.7: Let A and B be two fuzzy ideal of BCI algebra X with membership functionand respectively. A is contained in B if $\mu_A(x) \le \mu_B(x)$, $\forall x \in X$

Definition 2.9: An IFS A= < X, μ_A,υ_A> in a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:(F1) $\mu_A(0) \ge \mu_A(x)$ &$\upsilon_A(0) \ge \upsilon_A(x)$,(F2) $\mu_A(x) \ge \min\{\mu_A(x*y), \mu_A(y)\}$, (F3) $\upsilon_A(x) \le \max\{\upsilon A(x*y), \upsilon_A(y)\}$, for all x,y X.

Definition 2.10: An intuitionistic fuzzy set A=< μ_A, υ_A> of a BCI-algebra X is called an intuitionistic fuzzy a-ideal if it satisfies (F1) and(F4) $\mu_A(y*x) \ge \min\{\mu_A((x*z)*(0*y)), \mu_A(z)\}$,(F5) $\upsilon_A(y*x) \le \max\{\upsilon_A((x*z)*(0*y)), \upsilon_A(z)\}$, for all x,y,z X.

An interval-valued intuitionistic fuzzy set A defined on X is given byA={(x,[μ (x)μ (x)],[υ

μ ,μ are two membership functions and υ ,υ are two non-membership functions X such that μ ≤μ &υ ≥υ ,∀ x X. Let μ_A(x)=[μ ,μ]&υ (x)=[υ ,υ],∀ x X and let D[0,1]denote the family of all closed subintervals of [0,1].If μ (x)=μ (x)=c,0≤c≤1 and if υ (x)=υ (x)=k, 0≤k≤1,then we have $\mu_A(x)=[c,c]$&$\upsilon_A(x)=[k,k]$ which we also assume, for the sake of convenience, to belong to D[0,1]. thus $\mu_A(x)$&$\upsilon_A(x)$ [0,1],∀ x X,and therefore the i-v IFS a is given by A=[(x,$\mu_A(x)$,$\upsilon_A(x)$)},∀ x X,where $\mu_A(x)$:X→D[0,1]. Now let us define what is known as refined minimum, refined maximum of two elements in D[0,1].we also define the symbols" ≤","≥" and "=" in the case of two elements in D[0,1]. Consider two elements D_1:[a_1,b_1]and D_2:[a_2,b_2] D[0,1]. Then rmin(D_1,D_2)=[min{a_1,a_2},min{b_1,b_2}],
rmax(D_1,D_2)=[max{a_1,a_2},max{b_1,b_2}]
$D_1 \ge D_2 \Leftrightarrow a_1 \ge a_2, b_1 \ge b_2$; $D_1 \le D_2 \Leftrightarrow a_1 \le a_2, b_1 \le b_2$ and
$D_1 = D_2$.

III. INTERVAL-VALUED INTUITIONISTIC FUZZY A-IDEALS OF BCI-ALGEBRAS

Definition 3.1: An interval-valued intuitionistic fuzzy set A in BCI-algebra X is called an interval-valued intuitionistic fuzzy a-ideal of X if it satisfies (FI_1)$\mu_A(0) \ge \mu_A(x)$,$\upsilon_A(0) \le \upsilon_A(x)$,($FI_2$)$\mu_A(y*x) \ge r\min\{\mu_A((x*z)*(0*y)),\mu_A(z)\}$, (FI_3)$\upsilon_A(y*x) \le r\max\{\upsilon_A((x*z)*(0*y)),\upsilon_A(z)\}$.

Theorem 3.2 Let A be an i-v intuitionistic fuzzy a-ideal of X. if there exists a sequence {x_n} in X such that 11111 $\lim_{n\to\infty} \bar{\mu}_A(x_n)$ =[1,1], $\lim_{n\to\infty} \bar{\nu}_A(x_n)$ =[0,0] then $\mu_A(0)$=[1,1] and$\upsilon_A(0)$=[0,0].

Proof: Since $\mu_A(0) \ge \mu_A(x)$and $\upsilon_A(0) \le \upsilon_A(x)$ for all x X, we have $\mu_A(0) \ge \mu_A(x_n)$ and $\upsilon_A(0) \le \upsilon_A(x_n)$, for every positive integer n. note that $[\mu_A^L, \mu_A^U] \ge \bar{\mu}_A(0)$.[1,1] $\ge \mu_A(x) \ge \mu_A(0) \ge \lim_{n\to\infty} \bar{\mu}_A(x_n)$ (x_n)=[1,1]. $[\lambda_A^L, \lambda_A^U] \le \bar{\lambda}_A(0)$.[0,0] $\le \upsilon_A(x) \le \upsilon_A(0) \le \lim_{n\to\infty} \bar{\nu}_A(x_n)$ =[0,0].Hence $\mu_A(0)$=[1,1] and$\upsilon_A(0)$=[0,0].

Lemma3.3: An i-v intuitionistic fuzzy set A=[μ , υ] in X is an i-v intuitionistic fuzzy a-ideal of X if and only if and are intuitionistic fuzzy ideals of X.

Proof: Sinceμ (0) ≥μ (x); μ (0) ≥μ (x);υ (0) ≤ υ ( )and υ (0) ≤ υ ( ), therefore $\mu_A(0) \ge \mu_A(x)$, $\upsilon_A(0) \le \upsilon_A(x)$. Suppose that μ and υ are intuitionistic fuzzy ideal of X. let x,y X, then

(x),μ (x)] ≥[min{μ (x*y),μ (y)},min{μ (x*y),μ (y)}]

μ (y)]}

= r min {$\mu_A(x*y)$,$\mu_A(y)$} and

(x),υ (x*y),υ (y)},max{υ (x*y),υ (y)}]

= r max {$\upsilon_A(x*y)$,$\upsilon_A(y)$}.Hence A is an i-v intuitionistic fuzzy ideal of X.

Conversely, assume that A is an i-v intuitionistic fuzzy ideal of X. for any x,y X,we have

= [min {μ (x*y),μ (y)},min{μ (x*y),μ (y)}]

= [max {υ (x*y),υ (y)},min{υ (x*y),υ (y)}]

It follows thatμ (x) ≥ min {μ (x*y),μ (y)},υ (x) ≤ max{υ (x*y),υ (y)}

Andμ (x) ≥ min {μ (x*y),μ (y)},

Hence μ , μ and υ are intuitionistic fuzzy ideals of X.

Theorem 3.4. Every i-v intuitionistic fuzzy a-ideal of a BCI-algebraX is an i-v intuitionistic fuzzy ideal.
Definition 3.5: An i-v intuitionistic fussy set A in X is called an interval-valued intuitionistic fuzzy BCI-sub algebra of X if $\mu_A(x*y) \geq r\min\{\mu_A(x), \mu_A(y)\}$ and $\upsilon_A(x*y) \leq \{\upsilon_A(x), \upsilon_B(y)\}$, for all x,y X.

Proof: Let A=[μ , μ , υ , υ] be an i-v intuitionistic fuzzy a-ideal of X, where μ , μ and υ , υ are intuitionistic fuzzy a-ideal of X. thus μ , μ and υ , υ are intuitionistic fuzzy a-ideals of X. hence by lemma 3.3, A is i-v intuitionistic fuzzy ideal of X.

Theorem 3.6: Every i-v intuitionisticfuzzy a-ideal of a BCI-algebra X is an i-v intuitionistic fuzzy sub algebra of X.
Proof: Let A=[μ , μ , υ , υ]be an i-v intuitionistic fuzzy a-ideal of X, where μ , μ , and υ , υ are intuitionistic fuzzy a-ideal of BCI-algebra X. thus μ , μ , and υ , υ are intuitionistic fuzzy subalgebra of X. Hence, A is i-v intuitionistic fuzzysub algebra of X.

IV. CARTESIAN PRODUCT OF I-V INTUITIONISTIC FUZZY A-IDEALS

Definition 4.1 An intuitionistic fuzzy relation A on any set a is a intuitionistic fuzzy subset A with a membership function $\Omega_A: X\times X \to [0, 1]$ and non membership function $\Psi_A: X\times X \to [0, 1]$.

Lemma 4.2 Let μ_A and μ_B be two membership functions and υ_A and υ_B be two non membership functions of each x X to the i-v subsets A and B, respectively. Then $\mu_A\times \mu_B$ is membership function and $\upsilon_A\times \upsilon_B$ is non membership function of each element(x,y) X×X to the set A×B and defined by $(\mu_A\times\mu_B)(x,y)=r\min\{\mu_A(x), \mu_B(y)\}$ and $(\upsilon_A\times\upsilon_B)(x,y)=r\max\{\upsilon_A(x),\upsilon_B(y)\}$.

Definition 4.3 Let A= [μ , μ , υ , υ]and B=[μ , μ , υ , υ] be two i-v intuitionistic fuzzy subsets in a set X. the Cartesian product of A×B is defined by $A\times B= \{((x,y), (\mu_A\times\mu_B), (\upsilon_A\times\upsilon_B)); \forall x,y\ X\times X\}$ Where $A\times B: X\times X\to D[0,1]$.

Theorem 4.4. Let A=[μ υ]and B=[μ υ] be two i-v intuitionistic fuzzy subsets in a set X,then A×B is an i-v intuitionistic fuzzy a-ideal of X×X.

Proof: Let(x,y) X×X, then by definition

$(\mu_A\times\mu_B)(0,0)=r\min\{\mu_A(0), \mu_B(o)\} = r\min\{[\mu(0),\mu(0)],[\mu(0),\mu(0)]\}$

$=[\min\{\mu(0),\mu(0)\},\min\{\mu(0),\mu(0)\}]$

$\geq[\min\{\mu(x),\mu(y)\},\min\{\mu(x),\mu(y)\}]$

$=r\min\{[\mu(x),\mu(x)],[\mu(y),\mu(y)]\}$

$= r\min\{\mu_A(x), \mu_B(y)\}=(\mu_A\times\mu_B)(x,y)$

And $(\upsilon_A\times\upsilon_B)(0,0) =r\max\{\upsilon_A(0), \upsilon_B(o)$

$= r\max\{[\upsilon(0),\upsilon(0)],[\upsilon(0),\upsilon(0)]\}$

$=[\max\{\upsilon(0),\upsilon(0)\},\max\{\upsilon(0),\upsilon(0)\}] \leq[\max\{\upsilon(x),\upsilon(y)\},\max\{\upsilon(x),\upsilon(y)\}]$

$=r\max\{[\upsilon(x),\upsilon(x)],[\upsilon(y),\upsilon(y)]\}$

$= r\max\{\upsilon_A(x),\upsilon_B(y) =(\upsilon_A\times\upsilon_B)(x,y)$

Therefore (FI_2) holds.Now, for all x,y,z X, we have

$(\mu_A\times\mu_B)((y, y')*(x, x'))=(\mu_A\times\mu_B)(y*x, y'*x')$

$=r\min\{\mu_A(y*x),\mu_B(y'*x')\}$

$\geq r\min\{r\min\{\mu_A((x*z)*(0*y)), \mu_A(z)\}, r\min\{\mu_A((x^1*z^1)*(0*y^1)), \mu_A(z^1)\}\}$

$=r\min\{\{\min\{\mu^L{}_A((x*z)*(0*y)), \mu^L{}_A(z)\}, \min\{\mu^U{}_A((x*z)*(0*y)), \mu^U{}_A(z)\}\},$

$\{\min\{\mu^L{}_B((x^1*z^1)*(0*y^1)), \mu^L{}_B(z^1)\}, \min\{\mu^U{}_B((x^1*z^1)*(0*y^1)), \mu^U{}_B(z^1)\}\}$

$=\{\min\{\min\{\mu^L{}_A((x*z)*(0*y)), \mu^L{}_B((x^1*z^1)*(0*y^1))\}, \min\{\mu^L{}_A(z), \mu^L{}_B(z^1)\}\},$

$\min\{\min\{\mu^U{}_A((x*z)*(0*y)), \mu^U{}_B((x^1*z^1)*(0*y^1))\}, \min\{\mu^U{}_A(z), \mu^U{}_B(z^1)\}\}\}$

$=r\min\{(\mu_A\times\mu_B)(((x*z)*(0*y)), ((x^1*z^1)*(0*y^1))), (\mu_A\times\mu_B)(z, z')\}$

Also, $(\upsilon_A\times\upsilon_B)((y, y')*(x, x'))= (\upsilon_A\times\upsilon_B)(y*x, y'*x')$

$=r\max\{\upsilon_A(y*x),\upsilon_B(y'*x')\}$

$\leq r\max\{r\max\{\nu_A((x*z)*(0*y)),\nu_A(z)\}, r\max\{\nu_A((x^1*z^1)*(0*y^1)),\nu_A(z^1)\}\}$

$=r\max\{\{\max\{\nu^L{}_A((x*z)*(0*y)),\nu^L{}_A(z)\}, \max\{\nu^U{}_A((x*z)*(0*y)),\nu^U{}_A(z)\}\},$

$\{\max\{\nu^L{}_B((x^1*z^1)*(0*y^1)),\nu^L{}_B(z^1)\}, \max\{\nu^U{}_B((x^1*z^1)*(0*y^1)),\nu^U{}_B(z^1)\}\}$

$=\{\max\{\max\{\nu^L{}_A((x*z)*(0*y)),\nu^L{}_B((x^1*z^1)*(0*y^1))\},\max\{\nu^L{}_A(z),\ \nu^L{}_B(z^1)\}\},$

$\max\{\max\{\nu^U{}_A((x*z)*(0*y)),\nu^U{}_B((x^1*z^1)*(0*y^1))\},\max\{\nu^U{}_A(z),\nu^U{}_B(z^1)\}\}\}$

$=\ r\ \max\{(\nu_A\times\overline{\nu}_B)(((\overline{x}*z)*(0*y)),((x^1*z^1)*(0*y^1)),(\nu_A\times\nu_B)(z,z^1)\}$

Hence A×B is an i-v intuitionistic fuzzy a-ideal of X× X

Definition 4.5: Letμ_B,υ_B respectively, be an i-v membership and non membership function of each element x X to the set B. then strongest i-v intuitionistic fuzzy set relationon X ,that is a membership function relation $\mu_A$onμ_B and non membership function relation υ_A onυ_B and μ_{AB} ,ν_{AB} whose i-v membership and non membership function, of each element (x,y) X×X and defined by $\overline{\mu_{AB}}$(x,y)=r min{μ_B(x),μ_B(y)}&$\overline{\nu_{AB}}$ (x,y)=r max{υ_B(x),υ_B(y)}

Definition 4.6 Let B=[μ , μ , υ , υ] be an i-v subset in a set X, then the strongest i-v intuitionistic fuzzy relation on X that is a i-v A on B is A_B and defined by,A_B=[μ^L_{AB} , μ^U_{AB} , ν^L_{AB} , ν^U_{AB}].

Theorem 4.7 Let B=[μ^L_{AB} , μ^U_{AB} , ν^L_{AB} , ν^U_{AB}] be an i-v subset in a set X and A_B=[μ^L_{AB} , μ^U_{AB} , ν^L_{AB} , ν^U_{AB}] be the strongest i-v intuitionistic fuzzy relation on X. then B is an i-v intuitionistic a-ideal of X if and only if A_B is an i-v intuitionistic fuzzy a-ideal of X×X.

Proof: Let B be an i-v intuitionistic fuzzy a-ideal of X. thenμ_{AB}(0,0)=r min{μ_B(0),μ_B(0)} ≥r min{μ_B(x),μ_B(y)}=μ_{AB}(x,y) and υ_{AB}(0,0)=r max{υ_B(0),υ_B(0)}≤r max{υ_B(x),υ_B(y)}=υ_{AB}(x,y) ∀(x,y) X×X.

On the other hand $\overline{\mu}_{A_B}$ $((y_1,y_2)*(x_1,x_2))=\mu_{AB}(y_1*x_1,\ y_2*x_2)$

` $=r\min\{\mu_B(y_1*x_1),\mu_B(y_2*x_2)\}$

$\geq r\min\{r\min\{\mu_B((x_1*z_1)*(0*y_1)),\mu_B(z_1)\},r\min\{\mu_B((x_2*z_2)*(0*y_2)),\mu_B(z_2)\}\}$

$=r\min\{r\min\{\mu_B((x_1*z_1)*(0*y_1)),\mu_B((x_2*z_2)*(0*y_2))\},r\min\{\mu_B(z_1),\mu_B(z_2)\}\}$

$=r\min\{\mu_{AB}((x_1*z_1)*(0*y_1),(x_2*z_2)*(0*y_2)),\mu_{AB}(z_1,z_2)\}$

$=r\min\{\mu_{AB}(((x_1,x_2)*(z_1,z_2))*(0*(y_1,y_2))),\mu_{AB}(z_1,z_2)\}$

Also,$\overline{\nu}_{AB}$ $((y_1,y_2)*(x_1,x_2))=\upsilon_{AB}(y_1*x_1,\ y_2*x_2)$

$=r\max\{\upsilon_B(y_1*x_1),\upsilon_B(y_2*x_2)\}$

$\leq r\max\{r\max\{\upsilon_B((x_1*z_1)*(0*y_1)),\upsilon_B(z_1)\},r\max\{\upsilon_B((x_2*z_2)*(0*y_2)),\upsilon_B(z_2)\}\}$

$=r\max\{r\max\{\upsilon_B((x_1*z_1)*(0*y_1)),\upsilon_B((x_2*z_2)*(0*y_2))\},r\max\{\upsilon_B(z_1),\upsilon_B(z_2)\}\}$

$=r\max\{\upsilon_{AB}((x_1*z_1)*(0*y_1),(x_2*z_2)*(0*y_2)),\upsilon_{AB}(y_1,y_2)\}$ $=r\max\{\upsilon_{AB}(((x_1,x_2)*(z_1,z_2))*(0*(y_1,y_2))),\upsilon_{AB}(z_1,z_2)\}$

For all $(x_1,x_2),(y_1,y_2),(z_1,z_2)$ in X×X. hence A_B is an i-v intuitionistic fuzzy a-ideal of X×X.

Conversely, let A_B be an i-v intuitionistic fuzzy a-ideal of X×X. then for all (x,x) X×X.we have r min {μ_B(0),μ_B(0)}=μ_{AB}(0,0)≥μ_{AB}(x,x)= r min{μ_B(x),μ_B(x)}(or)μ_B(0) ≥μ_B(x) and r max {υ_B(0),υ_B(0)}=υ_{AB}(0,0)≤υ_{AB}(x, x)=rmin{υ_B(x),μ_B(x)}(or)υ_B(0)≤υ_B(x)∀x X. Now, let $(x_1,x_2),(y_1,y_2),(z_1,z_2)$ X×X, then rmin {$\mu_B(y_1*x_1),(y_2*x_2)$}=$\mu_{AB}(y_1*x_1,$

$y_2*x_2)=\mu_{AB}((y_1,y_2)*(x_1,x_2))$

$\geq r\min\{\mu_{AB}(((x_1,x_2)*((z_1,z_2))*(0*(y_1,y_2))),\mu_{AB}(z_1,z_2)\}=r$

$\min\{\mu_{AB}((x_1*z_1)*(0*y_1),(x_2*z_2)*(0*y_2)),\mu_{AB}(z_1,z_2)\}$

$=r\min\{r\min\{\mu_B((x_1*z_1)*(0*y_1)),\mu_B(z_1)\},r\min\{\mu_{AB}((x_2*z_2)*(0*y_2)),\mu_B(z_2)\}\}$ Also, rmax {$\upsilon_B(y_1*x_1),(y_2*x_2)$}=$\upsilon_{AB}(y_1*x_1,y_2*x_2)=\upsilon_{AB}((y_1,y_2)*(x_1,x_2))$

$\leq r\max\{\upsilon_{AB}(((x_1,x_2)*((z_1,z_2))*(0*(y_1,y_2))),\upsilon_{AB}(z_1,z_2)\}=r$

$\max\{\upsilon_{AB}((x_1*z_1)*(0*y_1),(x_2*z_2)*(0*y_2)),\upsilon_{AB}(z_1,z_2)\}$

$=r\max\{r\max\{\upsilon_B((x_1*z_1)*(0*y_1)),\mu_B(z_1)\},r\max\{\upsilon_{AB}((x_2*z_2)*(0*y_2)),\upsilon_B(z_2)\}\}$

If $x_2=y_2=z_2=0$, then r min {$\mu_B(y_1*x_1),\mu_B(0)$}≥ r min {r min {$\mu_B((x_1*z_1)*(0*y_1)),\mu_B(z_1)$},$\mu_B$(0)} and r max {$\upsilon_B(y_1*x_1),\upsilon_B(0)$}≥ r max {r max {$\upsilon_B((x_1*z_1)*(0*y_1),\upsilon_B(z_1)$},$\upsilon_B$(0)}

$\mu_B(y_1*x_1)\geq r\min\{\mu_B((x_1*z_1)*(0*y_1),\mu_B(z_1)\}$ and

$\upsilon_B(y_1*x_1)\geq r\max\{\upsilon_B((x_1*z_1)*(0*y_1),\upsilon_B(z_1)\}$.

Therefore B is i-v intuitionistic fuzzy a-ideal of X.

Theorem 4.8: Ifμ_A is a i-v intuitionistic fuzzy a-ideal of BCI-algebra X, then $\overline{\mu_{A_m}}$ is also i-v intuitionistic fuzzy a-ideal of BCI-algebra X

Proof: For all x, y, z ∈X

1. $\bar{\mu}_A(0) \geq \bar{\mu}_A(x),\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x)$. $[\bar{\mu}_A(0)]^m \geq [\bar{\mu}_A(x)]^m,\ [\bar{\nu}_A(0)]^m \leq [\bar{\nu}_A(x)]^m$

$\bar{\mu}_A(0)^m \geq \bar{\mu}_A(x)^m,\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x)^m$. $\bar{\mu}_{A^m}(0) \geq \bar{\mu}_{A^m}(x),\ \bar{\nu}_{A^m}(0) \leq \bar{\nu}_{A^m}(x)\ \forall x \in X$

2. $\bar{\mu}_A(y*x) \geq r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}$. $[\bar{\mu}_A(y*x)]^m \geq [r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}]^m$

$\bar{\mu}_A(y*x)^m \geq r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}^m$. $\bar{\mu}_{A^m}(y*x) \geq r\min\{\bar{\mu}_A((x*z)*(0*y))^m, \bar{\mu}_A(z)^m\}$

$\bar{\mu}_{A^m}(y*x) \geq r\min\{\bar{\mu}_{A^m}((x*z)*(0*y)) \bar{\mu}_{A^m}(z)\}$

3. $\bar{\nu}_A(y*x) \leq r\max\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}$. $[\bar{\nu}_A(y*x)]^m \leq [r\max\{\bar{\nu}_A((x*z)*(0*y)) \bar{\nu}_A(z)\}]^m$

$\bar{\nu}_A(y*x)^m \leq r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\}^m$. $\bar{\nu}_{A^m}(y*x) \leq r\max\{\bar{\nu}_A((x*z)*(0*y))^m, \bar{\nu}_A(z)^m\}$

$\bar{\nu}_{A^m}(y*x) \leq r\max\{\bar{\nu}_{A^m}((x*z)*(0*y)) \bar{\nu}_{A^m}(z)\}$

Theorem 4.9: If $\mu_A$is a i-v intuitionistic fuzzy a-ideal of BCI-algebra X, then $\bar{\mu}_{A\cap B}$ isalso a i-v intuitionistic fuzzy a-ideal of BCI-algebra X

Proof: For all x, y, z $\in$X

1. $\bar{\mu}_A(0) \geq \bar{\mu}_A(x),\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x)$ *and* $\bar{\mu}_B(0) \geq \bar{\mu}_B(x),\ \bar{\nu}_B(0) \leq \bar{\nu}_B(x)$

$\min\{\bar{\mu}_A(0), \bar{\mu}_B(0)\} \geq \min\{\bar{\mu}_A(x), \bar{\mu}_B(x)\}, \min\{\bar{\nu}_A(0), \bar{\nu}_B(0)\} \leq \min\{\bar{\nu}_A(x), \bar{\nu}_B(x)\}$

$\bar{\mu}_{A\cap B}(0) \geq \bar{\mu}_{A\cap B}(x),\ \bar{\nu}_{A\cap B}(0) \leq \bar{\nu}_{A\cap B}(x)$

2. $\bar{\mu}_A(y*x) \geq r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\},\ \bar{\mu}_B(y*x) \geq r\min\{\bar{\mu}_B((x*z)*(0*y)), \bar{\mu}_A(z)\}$

$\{\bar{\mu}_A(y*x), \bar{\mu}_B(y*x)\} \geq \{r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}, r\min\{\bar{\mu}_B((x*z)*(0*y)), \bar{\mu}_B(z)\}\}$

$\min\{\bar{\mu}_A(y*x), \bar{\mu}_B(y*x)\} \geq \min\{r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}, r\min\{\bar{\mu}_B((x*z)*(0*y)), \bar{\mu}_B(z)\}\}$

$\geq \min\{r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_B((x*z)*(0*y))\}, r\min\{\bar{\mu}_A(z), \bar{\mu}_B(z)\}\}$

$\bar{\mu}_{A\cap B}(y*x) \geq r\min\{\bar{\mu}_{A\cap B}((x*z)*(0*y)), \bar{\mu}_{A\cap B}(z)\}$

3. $\bar{\nu}_A(y*x) \leq r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\}, \bar{\nu}_B(y*x) \leq r\max\{\bar{\nu}_B((x*z)*(0*y)), \bar{\nu}_A(z)\}$

$\{\bar{\nu}_A(y*x), \bar{\nu}_B(y*x)\} \leq \{r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\}, r\max\{\bar{\nu}_B((x*z)*(0*y)), \bar{\nu}_B(z)\}\}$

If one is contained in the other

$\min\{\bar{\nu}_A(y*x), \bar{\nu}_B(y*x)\} \leq \min\{r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\}, r\max\{\bar{\nu}_B((x*z)*(0*y)), \bar{\nu}_B(z)\}\}$

$\bar{\nu}_{A\cap B}(y*x) \leq r\max\{\min\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_B((x*z)*(0*y))\}, \min\{\bar{\nu}_A(z), \bar{\nu}_B(z)\}\}$

$\bar{\nu}_{A\cap B}(y*x) \leq r\max\{\bar{\nu}_{A\cap B}((x*z)*(0*y)), \bar{\nu}_{A\cap B}(z)\}$

Theorem 4.10: If $\mu_A$is a i-v intuitionistic fuzzy a-ideal of BCI-algebra X, then $\bar{\mu}_{A\cup B}$ isalso a i-v intuitionistic fuzzy a-ideal of BCI-algebra X.

Proof: For all x, y, z $\in$X

1. $\bar{\mu}_A(0) \geq \bar{\mu}_A(x),\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x)$ *and* $\bar{\mu}_B(0) \geq \bar{\mu}_B(x),\ \bar{\nu}_B(0) \leq \bar{\nu}_B(x)$

$\min\{\bar{\mu}_A(0), \bar{\mu}_B(0)\} \geq \min\{\bar{\mu}_A(x), \bar{\mu}_B(x)\}, \min\{\bar{\nu}_A(0), \bar{\nu}_B(0)\} \leq \min\{\bar{\nu}_A(x), \bar{\nu}_B(x)\}$

$\bar{\mu}_{A\cup B}(0) \geq \bar{\mu}_{A\cup B}(x),\ \bar{\nu}_{A\cup B}(0) \leq \bar{\nu}_{A\cup B}(x)$

2. $\bar{\mu}_A(y*x) \geq r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\},\ \bar{\mu}_B(y*x) \geq r\min\{\bar{\mu}_B((x*z)*(0*y)), \bar{\mu}_A(z)\}$

$\{\bar{\mu}_A(y*x), \bar{\mu}_B(y*x)\} \geq \{r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}, r\min\{\bar{\mu}_B((x*z)*(0*y)), \bar{\mu}_B(z)\}\}$

$\max\{\bar{\mu}_A(y*x), \bar{\mu}_B(y*x)\} \geq \max\{r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_A(z)\}, r\min\{\bar{\mu}_B((x*z)*(0*y)), \bar{\mu}_B(z)\}\}$

$\geq \max\{r\min\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_B((x*z)*(0*y))\}, r\max\{\bar{\mu}_A(z), \bar{\mu}_B(z)\}\}$

If one is contained in the other

$r\min\{\max\{\bar{\mu}_A((x*z)*(0*y)), \bar{\mu}_B((x*z)*(0*y))\}, \max\{\bar{\mu}_A(z), \bar{\mu}_B(z)\}\}$

$\bar{\mu}_{A\cup B}(y*x) \geq r\min\{\bar{\mu}_{A\cup B}((x*z)*(0*y)), \bar{\mu}_{A\cup B}(z)\}$

3. $\bar{\nu}_A(y*x) \leq r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\},\ \bar{\nu}_B(y*x) \leq r\max\{\bar{\nu}_B((x*z)*(0*y)), \bar{\mu}_A(z)\}$

$\{\bar{\nu}_A(y*x), \bar{\nu}_B(y*x)\} \leq \{r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\}, r\max\{\bar{\nu}_B((x*z)*(0*y)), \bar{\nu}_B(z)\}\}$

$\max\{\bar{\nu}_A(y*x), \bar{\nu}_B(y*x)\} \leq \max\{r\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_A(z)\}, r\max\{\bar{\nu}_B((x*z)*(0*y)), \bar{\nu}_B(z)\}\}$ $\bar{\nu}_{A\cup B}(y*x) \leq r\max\{\max\{\bar{\nu}_A((x*z)*(0*y)), \bar{\nu}_B((x*z)*(0*y))\}, \max\{\bar{\nu}_A(z), \bar{\nu}_B(z)\}\}$

$\bar{\nu}_{A\cup B}(y*x) \leq r\max\{\bar{\nu}_{A\cup B}((x*z)*(0*y)), \bar{\nu}_{A\cup B}(z)\}$

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